International Conference on Reaction-Diffusion Equations and Spatial Dynamics in Mathematical Biology

12-16 Oct 2026
Clermont-Ferrand
France

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Which diffusion strategy is best for a prey and a predator moving with Fractional Laplacians ?

Elisa Affili
LMRS, Université de Rouen Normandie

We consider a predator and a prey moving with Fractional Laplacians, letting the predator and the prey free to choose the exponent characterising the distribution of the jump lengths. Which exponent should the predator use to maximise its chance to catch the prey ? And which one should the prey select to minimise the possibility of meeting the predator ? In this talk, we study a funtional that describes the chance of meeting taking into account both strategies. We focus in particular on the case of a large initial distance and we find ranges for the parameter of fractional diffusion corresponding to the maximum (for the predator) and the minimum (for the prey) in dependence of the choice of the other species. These findings rely on very fine estimates of the fundamental solution of the fractional heat equation.

Spectral stability of steady solutions in some shallow water models

Junsik Bae
Kyungpook National University

We consider some shallow water models that admit various steady solutions depending on the prescribed boundary conditions and bottom topography. In particular, discontinuous steady solutions arise when the flow undergoes a transition from a supercritical to a subcritical regime, whereas continuous transcritical solutions occur in transitions from subcritical to supercritical flow. We investigate the spectral stability of these steady flows. This talk is based on the ongoing project with L. Miguel Rodrigues (University of Rennes) and Nina Aguillon (Sorbonne University).

A reaction-diffusion system and its dual process

Nathanaël Boutillon
University of Vienna, Austria

This talk will be devoted to systems of reaction-diffusion equations with Allee effect. Those systems can model interacting species with a spatial structure. I will show how to construct a dual process to such systems, based on branching Brownian motions. The dual process will allow us to find spreading properties of reaction-diffusion systems with Allee effect. (Joint work with João Luiz de Oliveira Madeira)

From stochastic individual-based models to free-boundary Hamilton-Jacobi equations

Nicolas Champagnat
Inria Center of University of Lorraine

This is joint work with Sylvie Méléard (Ecole Polytechnique) , Sepideh Mirrahimi (University of Toulouse) and Chi Tran Viet (Inria Lille).

We study a stochastic branching model for a population structured by a quantitative phenotypic trait and subject to births, deaths, and mutations. In a regime of large population and small mutations, and in logarithmic scales of size and time, we derive a certain class of free boundary Hamilton-Jacobi equations with state constraints from the stochastic individual-based system. This goes beyond the classical Hamilton-Jacobi equations obtained from deterministic models by taking into account the possible extinction of the system in certain regions of the trait space. The proof is obtained by combining methods for the analysis of Hamilton-Jacobi equations and probabilistic tools from the theory of large deviations and branching processes.

Harnack-type inequalities and traveling waves for non-cooperative nonlocal diffusion systems

Bo-Sheng Chen
National Yang Ming Chiao Tung University

This work investigates the existence and qualitative properties of traveling waves for a class of non-cooperative nonlocal diffusive systems on complex networks, motivated by multi-group disease-transmission models. In this model, movement is described by an integral kernel that permits long-distance jumps (e.g., animal migration, air travel, seed dispersal), thereby capturing dispersal effects absent in classical random diffusion. Due to the lack of classical elliptic estimates for nonlocal diffusive operators, analyzing traveling wave solutions presents significant challenges. To overcome these difficulties, we establish a nonlocal diffusive Harnack-type inequality in coupled systems. This provides the essential estimates to establish the existence of a minimal wave speed for traveling waves. This is a joint work with Prof. Chang-Hong Wu.

A Dirichlet boundary control problem for a socio-economical regard for lake eutrophication

Ioana CHIRU
LISC - INRAE

An excessive phosphorus input in a lake can lead to its eutrophication : a disruption of the flora and fauna equilibrium inducing a cyanobacteria's proliferation, which can be toxic for humans. Beyond the environmental harm, the sanitary impact of this phenomenon motivate the need to better understand and prevent it.
The biological dynamic of the eutrophication phenomenon is modeled *via* a partial differential equations state system. To take into account the trade-off between benefits inducing phosphorus inputs and damages caused by cyanobacteria, the dynamic is part of an optimal control problem. The phosphorus inputs acting as the control variable and being delivered from a river discharging in the lake, we place ourselves in a Dirichlet boundary control problem. A previous work [1] defined and stated the well-posedness of the state system and the existence of a global solution of the optimal control problem.
With the idea to numerically approximate the solution, we define the adjoint problem of the state system, and prove the well-posedness of the problem coupling direct and adjoint system that induces the well-posedness of the optimal control problem. The existence of a solution uses the Schauder fixed-point theorem, and its uniqueness the Grönwall lemma combined with a lifting of the boundary condition.

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Estimation of System Parameters for Dynamical Systems through Emulator-Informed Deep Generative Models

Hyunwoo Cho
POSTECH

Differential equations (DEs) are crucial for modeling the evolution of natural or engineered systems. Traditionally, the parameters in DEs are adjusted to fit data from system observations. However, system inference for nonlinear dynamic models remains a significant challenge, particularly when the available data are noisy, sparse, partially observable, or independently collected at distinct time points from different subjects.

In this talk, we present emulator-informed deep generative models for estimating system parameters under such imperfect data settings. First, we introduce the Emulator-Informed Deep-Generative Model (EIDGM), designed to handle repeated cross-sectional (RCS) data. EIDGM integrates a physics-informed neural network-based emulator that immediately generates DE solutions and a Wasserstein generative adversarial network-based parameter generator that can effectively mimic the RCS data. We demonstrate that EIDGM can accurately capture parameter distributions for several dynamical systems and experimental datasets.

We then present the Simulation-based Generative Model for Imperfect Data (SiGMoID), which enables precise and robust inference for dynamic systems when the data are noisy, sparse, or partially observable. We demonstrate that SiGMoID quantifies data noise, estimates system parameters, and infers unobserved system components.

Together, these approaches show that emulator-informed deep generative models can be applied to a wide range of dynamical systems and provide a flexible framework for estimating parameter distributions and inferring system dynamics from limited and imperfect observations.

Bifurcation and Stability in Keller–Segel Systems

Beomjun Choi
Department of Mathematical Sciences, KAIST

I will discuss bifurcation and stability phenomena in Keller–Segel systems. Starting from spatially homogeneous equilibria, we study the emergence of nonconstant steady states as the chemotactic sensitivity crosses critical values determined by the spectrum of the underlying domain. I will then discuss general stability and long-time behavior for Keller–Segel equations, emphasizing the competition between diffusion and chemotactic aggregation. In the final part, I will turn to the parabolic–parabolic Keller–Segel system and explain some of the additional difficulties caused by the dynamics of the chemical concentration.

A Dirichlet boundary control problem for lake eutrophication

Éloïse Comte
INRAE

The constant growing of anthropogenic activities leads to a deterioration of water quality resources which is a major concern. In lakes, the accumulation of nutrients, as phosphorus, implies an algae development, including cyanobacteria, and thus a perturbation of the ecosystem : this is the eutrophication.
We propose an optimal control problem for the lake eutrophication, modeling the dynamics of phosphorus stock and cyanobacteria concentration with coupled nonlinear partial differential equations. The control is represented by the load of phosphorus from the watershed into the lake and is only defined on the lake shore, thus leading to a Dirichlet boundary control problem. We shift the control from the boundary into the PDE by using a lifting, as it is generally done for this class of control problem.
We discuss the originality of the model and detail the shifting of the control. We prove the well-posedness of the state problem and the existence of a global solution for the optimal control problem. The adjoint problem is provided and we present a coupled state-adjoint system which will lead to a uniqueness result of the solution of the optimal control problem.

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Center manifold and Hopf bifurcation for semilinear evolution equations in an invariant subspace : An integrated semigroup approach

Arnaud Ducrot
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In this lecture we discuss Hopf bifurcation for a class of partial differential equations of reaction-diffusion type with nonlinear advection and boundary conditions. The presence of the nonlinear advection term prevents a direct analysis in the natural phase space. To overcome this difficulty, we develop an approach based on non-densely defined operators and integrated semigroup theory, which allows us to reformulate the equation so that the advection term appears as a perturbation acting on a subspace Y of more regular functions.
Within this functional framework, we establish the associated spectral theory and construct an invariant spectral decomposition directly in Y , including an extension property for the spectral projectors. These key estimates enable us to prove the well-posedness of the problem and to establish the existence of a smooth local center manifold. As a special case, we derive a Hopf bifurcation result for the original nonlocal and nonlinear reaction-diffusion-advection equation.

Global existence for a cross diffusion system with different mobilities

Charles Elbar
CMAP - Ecole Polytechnique

We consider a cross diffusion system of two populations, often called the Busenberg-Travis system. The two species are transported by the same pressure gradient with Darcy's law, modeling overcrowding effect (populations tend to move away from regions of high pressure). However, their mobility is different : the first species moves with mobility 1, whereas the second moves with mobility ν. The difficulty to prove existence is to prove strong compactness of each densities, which we achieve with a variant of the div-curl lemma applied to evolution PDEs.

Fluctuations of fully pushed stochastic travelling waves

Raphaël Forien
INRAE - BioSP

We study a class of reaction-diffusion equations perturbed by space-time Wright-Fisher noise of the form

$$\partial_t u = \partial_{xx} u + f(u) + \sqrt{\frac{1}{N} u(1-u)}\dot{W},$$

where $u(t,\cdot) : \mathbb{R} \to [0,1]$ and $\dot{W}$ is space-time white noise. Such equations naturally arise as scaling limits of the frequency of an allele under frequency-dependent selection in a spatially structured population. We assume that space is one-dimensional, that the corresponding deterministic equation admits travelling wave solutions, and that these travelling waves are pushed. In the small noise limit ($N \to \infty$), we show that the solution to the perturbed equation remains close to a shift of the deterministic wave profile, and we show that, on suitable time scales, the shift process converges in distribution to Brownian motion with drift. This characterisation of the limiting behaviour of the shift holds for the whole fully pushed regime, as defined by Birzu et al. almost 10 years ago, which includes both bistable reaction terms and some (but not all) monostable reaction terms with pushed behaviour. Our result can be interpreted as saying that the stochastic wave "lags behind" the deterministic wave, but the dependence of the lag on the noise is in stark contrast with what has been established for the (pulled) stochastic FKPP equation.

This is joint work with Alison Etheridge, Thomas Hughes and Sarah Penington.

The Independence-Preserving Property and Planar Web Geometry

Yoshihiro Gyotoku
Graduate School of Mathematical Sciences, The University of Tokyo

In this presentation, planar web geometry is used to study transformations that preserve independence. The basic question is which distributions of two independent random variables make the transformed variables independent again. For real-analytic diffeomorphisms and product measures with positive continuous densities, this question reduces to an Abelian functional equation associated with a planar 4-web. When this web is nonsingular, its Abelian relations form a vector space of dimension at most three. This gives a unified approach to classical independence characterisations, including the Kac–Bernstein and Lukacs theorems and the Matsumoto–Yor property, as well as characterisations associated with quadrirational Yang–Baxter maps. The role of coordinatewise changes of variables and the resulting geometric viewpoint on the classification of independence-preserving transformations are also discussed.

Fluctuating hydrodynamics for a chain of nonlinear stochastic oscillators

Kohei Hayashi
Graduate School of Science, The University of Osaka

We study fluctuations of phonon modes in a one-dimensional chain of anharmonic oscillators where the deterministic Hamiltonian dynamics is perturbed by random exchanges of momentum between nearest neighbor particles. There are three locally conserved quantities : volume, momentum and energy. We study the evolution in equilibrium of the fluctuation fields of the two phonon modes (linear combination of the volume stretch and momentum), on a diffusive space-time scale after recentering on their sound velocities. We show that, weakening the anharmonicity with the scale parameter, the recentered phonon fluctuations fields converge to the stationary solutions of two uncoupled stochastic Burgers equations. The nonlinearity in the Burgers equation depends on the presence of a cubic term in the anharmonic potential (corresponding to the α-FPUT dynamics). This talk is based on a recent joint work with Stefano Olla (Université Paris Dauphine - PSL, GSSI).

Fast reaction limit of some competition-diffusion systems arising in biology

Danielle Hilhorst
CNRS and University Paris-Saclay

The starting point is a competition-diffusion system which was proposed by Professor Masayasu Mimura. This system models two biological populations which compete for their habitat. In [1], we studied its limit as the reaction coefficient tends to zero and proved that it converges to a Stefan problem with zero latent heat. More recently, we revisited this question in two specific directions :

In collaboration with Hideki Murakawa, we consider the case that the linear diffusion terms are replaced by nonlinear degenerate diffusion ones. Here again, we present a priori estimates and pass to the limit as the reaction coefficient tends to infinity. In a special case, we also derive an error estimate.

In collaboration with Perla El Kettani and Kunwoo Kim, we study the fast reaction limit of a stochastic reaction-diffusion system and prove that its solution converges to the unique solution of a stochastic Stefan problem with zero latent heat as the reaction coefficient tends to infinity.

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On the existence and regularity of weakly nonlinear stationary Boltzmann equations : a Fredholm alternative approach

Chun-Hsiung Hsia
National Taiwan University

This is joint work with I-Kun Chen and Daisuke Kawagoe. The celebrated Fredholm alternative theorem works for the setting of identity compact operators. This idea has been widely used to solve linear partial differential equations. In this talk, we demonstrate a generalized Fredholm theory in the setting of identity power compact operators, which was suggested in 1987 by Cercignani and Palczewski to solve the existence of the stationary Boltzmann equation in a slab domain. We carry out the detailed analysis based on this generalized Fredholm theory to prove the existence theory of the stationary Boltzmann equation in bounded three-dimensional convex domains. To prove that the integral form of the linearized Boltzmann equation satisfies the identity power compact setting requires the regularizing effect of the solution operators. Once the existence and regularity theories for the linear case are established, with suitable bilinear estimates, the nonlinear existence theory is accomplished.

Validated Numerics for Reaction-Diffusion Equations: From Steady States to Solution Manifolds

Olivier Hénot
National Taiwan University

Computer-assisted proofs have become an essential tool in the study of nonlinear differential equations. In this talk, we will first present the general structure of a posteriori validation methods based on the contraction of a fixed-point operator. The contraction yields the existence and local uniqueness of a solution near a numerical approximation, with an explicit error bound.

We will then apply this framework to steady states of reaction-diffusion equations with linear and nonlinear diffusion on a bounded interval with Neumann boundary conditions. Examples include the Cahn–Hilliard equation and the Shigesada–Kawasaki–Teramoto cross-diffusion system.

Finally, we will show how the same argument extends to solution manifolds, that is, to families of steady states over a whole parameter region. The manifold is approximated by a high-order Chebyshev interpolation polynomial in the parameters, and the contraction is established in a weighted ℓ^1 space of Chebyshev coefficients.

When vector abundance constrains vector-borne transmission : inside epidemic dynamics within a prescribed population

Lise Isnel, Julien Papaïx, Samuel Soubeyrand, Lionel Roques
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Classical mechanistic models are typically formulated as closed dynamical systems, where every state variable evolves according to internally prescribed equations. In contrast, we adopt a prescriptive approach : empirical measurements are prescribed as an exogenous forcing directly into the governing differential equations. The resulting system is thus non-autonomous, and continuously driven by data.

We instantiate this framework on compartmental models describing vector-borne disease transmission. The total vector abundance, a state variable, is taken as an externally prescribed, non-negative input. We establish rigorous theoretical guarantees for introducing such a forcing term in a host-vector model, raising natural questions about well-posedness, and provide epidemiological results regarding *Aedes albopictus*-borne diseases in mainland France.

Keywords mathematical epidemiology, compartmental model, non-autonomous dynamical system, inter-seasonal transmission, entomological surveillance

Network conditions for global existence of reaction-diffusion equations generated by mass-action reaction networks

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We study reaction-diffusion systems arising from two-species chemical reaction networks (CRNs) under mass-action kinetics. When species are allowed to diffuse in a spatial domain, a CRN gives rise to a system of the form

$$\begin{aligned} \partial_t u &= d_u \Delta u + f(u,v), \\ \partial_t v &= d_v \Delta v + g(u,v), \end{aligned}$$

where $d_u > 0$, $d_v > 0$ are diffusion coefficients and f and g are the mass-action reaction terms. The well-posedness of such systems depends on sign conditions on the reaction terms f and g at the boundary of an invariant rectangle $[m_X, M_X] \times [m_Y, M_Y]$. We treat two cases according to the nature of the threshold : when the lower bounds are strictly positive($m_X, m_Y > 0$), pointwise sign conditions at the boundary of the rectangle suffice ; when the lower bounds vanish ($m_X = m_Y = 0$), the quasi-positivity of mass-action kinetics yields a linear factorization of the reaction terms, which replaces the sign conditions and allows a maximum principle argument to uniform estimate of the solutions.

The central contribution of this paper is to show that these analytical conditions are characterized by the geometric structure of the underlying CRN : specifically, we prove that if the network is weakly reversible and the highest-degree coefficient of f in X is necessarily negative, then this implies the required sign conditions and thereby guarantees global existence of classical solutions, together with an invariant region property

$$m_X < X(r,t) < M_X, \quad m_Y < Y(r,t) < M_Y \quad \text{for all } (r,t) \in (0,L) \times (0,\infty).$$

This connects the combinatorial geometry of the reaction network directly to the PDE analysis.

This work addresses a largely unexplored intersection of CRN theory and PDE analysis. Prior work of Tang and Morgan [1] has established global existence for reaction-diffusion systems arising from CRNs via entropy methods and Lyapnov-based intermediate sum conditions, primarily from the

PDE perspective without exploiting network geometry. On the ODE side, Craciun [2] has developed rich stability theory for spatially homogeneous CRN dynamics in the direction of the Global Attractor Conjecture. However, the addition of diffusion can fundamentally alter this picture[3], as blow-up examples in the spirit of Rothe's work on dissipative reaction-diffusion systems illustrate. Our results provide sharp network-theoretic conditions under which the PDE system remains globally well-posed, bridging the structural theory of CRNs and the regularity theory of parabolic systems.

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Reaction Network Approaches for Reaction-Diffusion Systems : Structural Conditions for Global Existence

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The global existence of the solutions to reaction-diffusion systems is widely studied classical problem. However, such studies are based on entropy inequality or mass-control, leading the degree limitations in reaction terms. [1, 2, 3]
In this paper, we present structural conditions of the global solution existence governed by chemical reaction network which do not depend on linearity nor parameters showing the rectangular forward invariant region in phase plane. In particular, we show that these structural classes are mainly constituted by weakly reversible networks involving birth and death process. Under these structural conditions, we show local existence and derive uniform estimate, using the maximum principal ensured by rectangular forward invariant region.
Moreover, we highlight the rectangular shape of forward invariant region motivated by prior study [4] of Pierre and Schmitt in 2000, describing the possibility to blow up under the other shape of invariant region.

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Dispersal and Dormancy Strategies in Temporally Fluctuating Environments

Yong-Jung Kim
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This study investigates, through mathematical models, how temporally varying environments shape biological traits. We focus on two traits : dispersal rate and the rate of activation from dormancy.

The first trait is the dispersal rate. When resources become scarce or environmental conditions deteriorate, migration is one possible strategy for survival. Among various forms of movement, we consider random dispersal, which is modeled by diffusion. A fundamental question is whether a larger or smaller dispersal rate is advantageous, or whether there exists an optimal dispersal rate. When such an optimum exists, we investigate how it is determined by the temporal variation of the environment.

A contrasting strategy is dormancy. Instead of moving, organisms may reduce energy consumption and enter a dormant state until environmental conditions improve. Once conditions become favorable, they return to an active state and resume growth and reproduction. We consider two types of dormancy mechanisms : a random mechanism that does not respond directly to environmental changes and an environmentally responsive mechanism. Our second objective is to determine the optimal activation rate from dormancy and to understand how it depends on the amplitude and frequency of environmental fluctuations.

Although dispersal and dormancy represent biologically contrasting strategies, both problems can be analyzed within the same class of ODE models. The population is divided into two states, with individuals switching between them at prescribed transition rates. Examples include hungry and satiated states, juvenile and adult stages, or dormant and active states. The dynamics of the entire population are then described through the interaction of these two subpopulations, each governed by its own dynamics. This common two-state framework allows us to examine how temporal environmental variability determines optimal transition-related traits across different biological contexts.

Blocking of Bistable Front Propagation : Application to Wolbachia Population Replacement

Saoussen Latrach
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In order to control epidemics of mosquito-borne diseases such as dengue, for which there is still no vaccine available, several strategies aim to directly target the mosquito population. One technique involves introducing a symbiotic bacterium called Wolbachia into the population, which blocks the transmission of pathogens. A mathematical model of the introduction of this bacterium and the replacement of the mosquito population with a population carrying this bacterium has been developed, proving that in a homogeneous environment, there is spatial propagation of the replacement. The aim of this work is to study the blocking of this propagation in cases where the environment is not homogeneous. In particular, we are considering scenarios where the environmental carrying capacity varies significantly across different regions of the spatial domain. Numerical simulations have been performed to validate the theoretical results.

On Graph scheme and its applications to a singularly perturbed problem.

Min-Gi Lee
Kyungpook National University

We propose a scheme to solve numerically a reaction-diffusion equation in two or three space dimensions. For a linear elliptic equation, we consider a formulation having the flux J and the pressure p as unknown functions. The corresponding well-posedness theory is available in the space

$$\mathsf{H}(\mathsf{div}\,,\Omega) \times \mathsf{H}^1(\Omega).$$

In short, our scheme is a discrete analogue of this formulation, in the space

$$\text{Raviart-Thomas}^0(\Omega) \times \mathbb{P}^1(\Omega),$$

without dual mesh. It is known that the mixed FEM or Finite Element Exterior Calculus is formulated in the space $\text{Raviart-Thomas}^0(\Omega) \times \mathbb{P}^0(\Omega)$ without dual mesh, and that Discrete Exterior Calculus is formulated in the space $\text{Raviart-Thomas}^0(\Omega) \times \mathbb{P}^1(\Omega)$ but using the dual mesh.

We apply the scheme to a singularly perturbed problem with a small parameter ϵ and illustrate the robustness of the scheme by tracking the rate of convergence or divergence to ϵ by numerical computations that are anticipated by a priori estimates.

Biological Interfaces and Reaction–Diffusion Dynamics: Variational Models and Constitutive Learning

Min-Jhe Lu
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The shape of a growing tumor and the spacing of intracellular condensates pose a common mathematical question: how do constitutive assumptions determine observable spatial patterns? Balance laws constrain evolution, but predictions also depend on the laws governing transport, growth, and mechanical response.

I first discuss free-boundary tumor models in which interfacial energy and bulk dissipation specify mechanical response. Stability analysis and nonlinear simulations quantify how nutrient supply and chemotaxis affect morphology [1]; related results address bifurcation structure and curvature-dependent stiffness. For chromosomal passenger complex condensates during cell division, numerical droplet calculations in a Cahn–Hilliard model connect measured focus sizes to an effective interfacial length. Condensate spacing then provides a separate observable for testing the calibrated model [2].

This distinction between balance laws and constitutive response also motivates a learning strategy for reaction–diffusion systems, including coupled species. The balance laws remain explicit, while the constitutive responses are learned. In diffusion and phase-separation tests, known laws supply targets on independently sampled fields, without generating training trajectories [3]. A separate fixed-grid analysis of linearized dynamics shows that lagging the response in a matched Heun comparison reduces temporal accuracy from second to first order in the reconstructed density. The analysis identifies how response evaluation must be coupled to state updates to retain second-order accuracy in this setting.

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Nontrivial Solutions of Bistable Reaction-Diffusion Equations on Finite-Length Graphs : Existence and Stability

Harunori Monobe
Osaka Metropolitan University

In this study, we investigate the existence and stability of steady-state solutions to reaction-diffusion equations with bistable nonlinearities on finite-length graphs. We first focus on a “star graph” with a single junction and establish the existence and stability of non-constant steady-state solutions when the edges are sufficiently long. We then extend these results to demonstrate the existence and stability of non-constant steady-state solutions on more general graph structures. This is joint work with Professor Yoshihisa Morita.

Propagation and blocking in reaction-diffusion equations on metric graphs

Hiroshi Matano
Meiji University

Reaction-diffusion equations on metric graphs are receiving growing attention these days. Roughly speaking, a metric graph is a directed graph whose edges have a length scale. In this lecture, I will talk about a bistable reaction-diffusion equation on a metric graph of rather an arbitrary configuration, and discuss the propagation and blocking of the solution fronts.

More precisely, the equation is defined on a metric graph Ω that consists of a bounded finite metric graph D of an arbitrary configuration (which I call the *center graph*), and a finite number of branches $\Omega_1, \dots, \Omega_N$ $(N \geq 2)$ of infinite length emanating from some of the vertices of D. Each Ω_i $(i = 1, \dots, N)$ is called an "outer path". Our goal is to investigate the behavior of the front coming from infinity along a given outer path Ω_i and to discuss whether or not the front propagates into other outer paths Ω_j $(j \neq i)$. Unlike the case of star graphs, where D is a single vertex, the dynamics of solutions can be far more complex and may depend sensitively on the configuration of the center graph D.

I first focus on general principles that apply to a large class of metric graphs, then I use this general theory to discuss the dynamics of specific examples. This is joint work with Shuichi Jimbo of Hokkaido University.

Bifurcation Analysis of Traveling Spot Solutions in a Two-Dimensional Free Boundary Problem

Natsume Motohashi
Graduate School of Science, Hokkaido University

In this talk, we consider a two-dimensional free boundary problem arising as the singular limit of a reaction-diffusion system with a nonlocal term. The reaction-diffusion system was originally proposed as a model for an alcohol droplet exhibiting spontaneous self-propelled motion on a water surface. Numerical studies have revealed two qualitatively different types of dynamics : droplet-like motion in some parameter regimes and solid-like motion in others. Despite these intriguing numerical observations, their underlying mathematical mechanisms and parameter dependence remain largely unexplored analytically.

Here, we focus on the onset of motion of localized spot patterns, particularly the emergence of traveling spot solutions from radially symmetric standing spots. We analyze the stability of the standing spots and show that they undergo a drift bifurcation. We then establish the existence of a branch of traveling spot solutions bifurcating from the standing spots and derive an explicit criterion that determines whether the resulting pitchfork bifurcation is supercritical or subcritical. Moreover, we develop a numerical bifurcation scheme and perform numerical bifurcation analysis, which confirms the analytical predictions and demonstrates the scheme's effectiveness.

On a reaction-diffusion model for the self-propelled motion of a rigid body

Masaharu Nagayama *

Although many mathematical models of self-propelled motion have been proposed, they are typically constructed for specific types of objects, resulting in separate formulations for different target systems. To date, no mathematical model has been established that can uniformly describe self-propelled motion across a wide range of systems, from solid objects to liquid droplets. As a first step toward a comprehensive theoretical understanding of self-propelled motion, it is therefore essential to formulate a unified set of equations capable of describing such phenomena. In this study, we propose the following mathematical model that could describe self-propelled motion from liquid droplets to solid objects within a single framework.

$$\begin{cases} \varepsilon^2\tau\dfrac{\partial\varphi}{\partial t} = \varepsilon^2\sigma^2\triangle\varphi + \varphi(1-\varphi)(\varphi - a(S[\varphi](t), u, \varepsilon)), \\ \dfrac{\partial u}{\partial t} = \triangle u - ku + \varphi, \end{cases} \qquad \boldsymbol{x}\in\Omega, \quad t>0, \tag{1}$$

where

$$\begin{aligned} a(S[\varphi](t), u, \varepsilon) &:= \frac{1}{2} + \varepsilon(-\gamma(u) + S[\varphi](t)), \\ \gamma(u) &= \frac{1}{1+(u/u_1)^m} + \gamma_0, \\ S[\varphi](t) &= \alpha\left(\int_\Omega \varphi(\boldsymbol{x}, t)\, d\boldsymbol{x} - S_0\right). \end{aligned}$$

The model system (1) consists of a coupled system of a phase-field equation, derived from the L^2 gradient-flow structure, and a concentration-field equation. This formulation enables the description of deformable droplet motion as well as circular solid-like motion. Furthermore, by introducing a spatially inhomogeneous function into the potential term, the model can reproduce self-propelled motions with more complex shapes, such as elliptical and dumbbell-like structures. In this presentation, we investigate solid-like motions observed in the proposed model through numerical simulations and compare the results with those of previous studies and experimental observations.

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Linear versus nonlinear determinacy of the minimum speed of traveling waves in diffusive Lotka-Volterra competition models

Ken-Ichi Nakamura
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Monostable traveling waves in reaction-diffusion systems describe critical phenomena such as the invasion of non-native species and the spatial spread of infectious diseases. Determining their minimum propagation speed is essential from both mathematical and ecological perspectives. While the minimum speed for single reaction-diffusion equations can be characterized using phase plane analysis, extending this methodology to systems with multiple components presents significant challenges. In this talk, we discuss the linear versus nonlinear determinacy of the minimum speed in the classical Lotka-Volterra competition-diffusion system by utilizing appropriate supersolutions and subsolutions under the comparison theorem.

Entropic metastability and the narrow escape problem: a quasi-stationary distribution approach

We study quasi-stationary distributions (QSD) for entropic metastability, modeled by a drifted, reflected diffusion on a bounded domain with mixed absorbing-reflecting boundary conditions, the setting of the narrow escape problem at fixed window size. We prove existence and uniqueness of the QSD under minimal regularity assumptions, then identify its density with the principal eigenfunction of the adjoint generator and establish its regularity up to the interface between the absorbing and reflecting parts of the boundary, where these two regions meet tangentially, a regime in which solutions to the associated mixed boundary value problem generically lose derivative regularity. We deduce the exit position distribution from the quasi-stationary regime.

Joint work with Louis Carillo and Loïs Delande.

Constraint-Augmented Physics-Informed Neural Networks for Parameter Calibration of Reaction–Diffusion Equations

Jayrah Bena E. Riñon
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University of the Philippines Diliman

Physics-informed neural networks (PINNs) have emerged as a powerful framework for solving inverse problems governed by partial differential equations. In this work, we study the calibration of reaction–diffusion equations from observational data using PINNs. We show that standard PINN formulations are generally insufficient to recover the complete set of model parameters and that additional constraints are necessary to ensure parameter identifiability. Motivated by this observation, we propose a constrained PINN formulation that augments the conventional loss function with additional physically motivated constraints. We provide a theoretical analysis of the proposed method by proving consistency and stability theorems. These results offer a rigorous foundation for constrained PINN-based calibration and clarify the role of additional constraints in guaranteeing reliable parameter recovery. The effectiveness of the proposed framework is demonstrated through numerical experiments on several benchmark reaction–diffusion models, including the one-dimensional and two-dimensional Fisher–KPP equations and the Allen–Cahn equation. The numerical results confirm the theoretical findings and show that the proposed constrained formulation accurately recovers the model parameters while maintaining high-fidelity solution reconstruction.

Stationary solutions for the heterogeneous Fisher–KPP equation

Luca Rossi
Università di Roma "Sapienza"

Does the heterogeneous Fisher-KPP equation admit a unique stationary solution ? We will present a positive answer in dimension at most 6. Then we will see that some surprises may occur in higher dimension. These results shed light on the question of long-time convergence for solutions of the reaction-diffusion equation.

This is a joint work with O. Tough.

When can a population spreading across sink habitats persist ?

Edouard Strickler
CNRS, Université de Lorraine, Inria

We consider populations with time-varying growth rate living in sinks. Each population, when isolated, would become extinct. Disperal-Induced Growth (DIG) occurs when the populations are able to persist and grow exponentially when dispersal among the habitats is present. We give sufficient and necessary conditions for DIG to happen. We present two approaches, one based on the study of the asymptotics of Lyapunov epxonent, another one based on elementary minorisation along migration paths. Based on joint works with Michel Benaïm, Claude Lobry and Tewfik Sari

Neural operators for system of reaction-diffusion equations

Jenn-Nan Wang
Department of Mathematics, National Taiwan University, Taiwan

In this talk, I would like to discuss a neural operator framework for learning the solution operators of reaction-diffusion systems. The proposed architecture incorporates spatial and temporal coordinate information to capture location-dependent reaction dynamics and provides a flexible representation for nonlinear evolution operators. We establish a universal approximation theorem, showing that the proposed neural operator can approximate a broad class of reaction-diffusion solution operators arbitrarily well. Furthermore, we derive quantitative approximation estimates that characterize the relationship between approximation accuracy and network complexity under suitable regularity assumptions. Numerical experiments on representative reaction-diffusion systems demonstrate the effectiveness of the proposed method in accurately learning complex solution dynamics and generalizing to unseen initial conditions. These results provide both theoretical and computational foundations for neural operator learning of reaction-diffusion equations. This talk is based on the joint work with Furuya and Ozawa.

Bramson correction of spreading fronts in shifting environments

Chang-Hong Wu
National Yang Ming Chiao Tung University

This talk examines the asymptotic behavior of solutions to the Fisher-KPP equation on the real line in a heterogeneous environment characterized by a moving jump discontinuity. We determine the location of the spreading front and establish convergence to a traveling wave profile. Our analysis relies on an extension of Bramson's classical results to equations with expanding domains and a gluing technique to construct precise super- and subsolutions. This is joint work with King-Yeung Lam (The Ohio State University).

Traveling waves in a state-dependent switching model of skin eruptions

Ying Xie
Kyoto University

Chronic spontaneous urticaria (CSU) is a skin disorder characterized by wheals that can expand over the skin and form diverse spatial eruption patterns. In recent work, Seirin-Lee and collaborators developed a reaction–diffusion–ODE model that numerically reproduces several characteristic eruption morphologies, while the mathematical mechanism underlying their spatial propagation remained unclear. Our recent analysis indicates that traveling waves of a reduced system play a central role in this propagation. We consider

$$u_t = Du_{xx} + g(u, I), \qquad I_t = u,$$

where

$$g(u, I) = h(u)\chi(I/\bar{H}) - \mu u, \qquad \chi(s) = \begin{cases} 1, & s \le 1, \\ 0, & s > 1, \end{cases}$$

and $f(u) := h(u) - \mu u$ is bistable. The variable I records the accumulation of u and hence carries its history. When I crosses the threshold $\bar{H}$, the reaction changes discontinuously from $f(u)$ to $-\mu u$, corresponding to a switch from an active positive-feedback regime to a linear decay regime. Thus, the model exhibits state-dependent discontinuous switching triggered by a history-dependent variable.

We study the resulting switched pulse waves, consisting of a bistable semi-wave in the active regime joined to an exponentially decaying branch in the decay regime. For each prescribed speed $0 < c < c^*$, we establish the existence and uniqueness of the switched pulse profile. To represent a traveling wave of the full history-dependent system, the profile must additionally satisfy the threshold condition

$$\int_{z_{\mu,c}}^{\infty} \phi_c(z)\, dz = c\bar{H},$$

which defines an admissible switched pulse wave. I will discuss the existence of such admissible waves and how their speed and height depend on model parameters and the threshold $\bar{H}$. This is joint work with Hiroshi Matano (Meiji University) and Sungrim Seirin-Lee (Kyoto University).

Nonlinear motility in biological reaction-diffusion systems

Changwook Yoon
Chungnam National University

This talk concerns nonlinear movement mechanisms arising in several biological reaction-diffusion models. We consider density-dependent motility in chemotaxis-consumption systems, ratio-dependent motility motivated by starvation-driven movement, and prey-taxis models incorporating crowding and prey-dependent dispersal. For these models, we discuss mathematical difficulties caused by nonlinear diffusion and taxis terms and present analytical approaches for deriving suitable a priori estimates. Particular emphasis is placed on global existence and boundedness of solutions and on how different biological movement mechanisms affect the mathematical structure of the corresponding PDE systems.